Finite Element Modeling of Magnetoelectric Composites with Interdigitated Electrodes

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Magnetoelectric effect in composite materials results from the combination of piezoelectric and magnetostrictive effects via elastic interaction. This work focuses on the modeling of magnetoelectric multilayer structures with interdigitated electrodes. A finite element formulation for such coupled problems, taking into account the nonlinearity of magnetostrictive material, and an application to a magnetic sensor are presented.

Index Terms—**Magnetoelectric effect, interdigitated electrodes, finite element formulation, magnetostriction, piezoelectricity.**

I. Introduction

THE study of magnetoelectric (ME) composite is a long-standing domain of interests in physics due to the possibilities of applications such as sensors, actuators, memory devices.... The magnetoelectric phenomenon consists in the existence of an electric polarization induced by a magnetization or, conversely, a magnetization induced by an electric polarization. The magnetoelectric composite materials, made of the assembly of magnetostrictive and piezoelectric constituents, have shown to exhibit the highest extrinsic ME effect than homogeneous intrinsic materials [1]. A significant and additional factor when determining the ME performance is the geometry of the electrodes associated to the piezoelectric material. Conventional electrodes bonded on surfaces of a piezoelectric material use the electric field through the material thickness and the transverse (d_{31}) piezoelectric coefficient. It results a planar actuation due to the electric fields through the thickness of the film and the transverse piezoelectric effect. Other electrode configurations are the interdigitated electrode (IDE) patterns. IDE [2] consist of a series of opposing polarity electrodes deposited on either side of a piezoelectric material. By using IDE, a large component of the electric field can be aligned in the direction of actuation and polarisation. Thus, the longitudinal (d_{33}) piezoelectric coefficient is utilized in this direction. As the longitudinal piezoelectric effect can be significantly larger than the transverse effect ($d_{33}/d_{31} \approx 2.5$ for most of PZT) an increased planar actuation is obtained, leading to an increase of the magnetoelectric coupling.

In this work a model based on finite element method is developed for magnetoelectric composites with interdigitated electrodes. In a first part, the formulation based on a thermodynamical approach is introduced. The model is then applied to a magnetoelectric sensor.

II. Constitutive Laws

A. Piezoelectric and magneto-elastic behaviours

The behaviour of active materials, when the loss are neglected, is given by the knowledge of the dependence of the electric flux density $\mathbf{D}$ and the stress tensor $\mathbf{T}$ to the electric field $\mathbf{E}$ and the strain tensor $\mathbf{S}$ for piezoelectric materials, and of the magnetic field $\mathbf{H}$ and the stress $\mathbf{T}$ to the magnetic induction $\mathbf{B}$ and the strain $\mathbf{S}$ for magnetostrictive materials.

$$\mathbf{T}(\mathbf{E},\mathbf{S}) \quad \mathbf{D}(\mathbf{E},\mathbf{S}) \qquad (1) \qquad\qquad \mathbf{T}(\mathbf{B},\mathbf{S}) \quad \mathbf{H}(\mathbf{B},\mathbf{S}) \qquad (2)$$

Based on thermodynamic potential and energy exchange, the definition of (1) and (2) requires the use of piezoelectric coefficients $\mathbb{a}$ [3] as well as piezomagnetic coefficients $\mathbb{g}$ [4]:

$$\alpha_{ikl} = \frac{\partial d_i}{\partial s_{kl}} = -\frac{\partial t_{kl}}{\partial e_i} \quad (3) \qquad\qquad \gamma_{ikl} = \frac{\partial h_i}{\partial s_{kl}} = \frac{\partial t_{kl}}{\partial b_i} \quad (4)$$

Considering that piezoelectric materials are usually prepolarized, the piezoelectric constitutive law is assumed to be linear around the polarization point:

$$\begin{pmatrix} \mathbf{T} \\ \mathbf{D} \end{pmatrix} = \begin{bmatrix} \mathbb{c}^e & -\mathbb{a}^t \\ \mathbb{a} & \mathbb{e}^s \end{bmatrix} \begin{pmatrix} \mathbf{S} \\ \mathbf{E} \end{pmatrix} \qquad (5)$$

where $\mathbb{c}^e$ is the stiffness tensor at constant electric field and $\mathbb{e}^s$ is the electrical permittivity at constant strain.

Unlike the piezoelectric material, the magnetostrictive material is not prepolarized. Its behaviour is highly non linear and this non linearity has to be considered in the magnetostrictive constitutive laws. The mechanical behaviour is written in the framework of linear elasticity, using the decomposition of the total strain into an elastic strain $\mathbf{S}^e$ and the magnetostriction strain $\mathbf{S}^\mu$ ($s_{kl} = s^e_{kl} + s^\mu_{kl}$):

$$t_{ij}(\mathbf{B},\mathbf{S}) = C^{ms}_{ijkl}(s_{kl} - s^\mu_{kl}(\mathbf{B})) \qquad (6)$$

with C^{ms}_{ijkl} the usual stiffness tensor of the magnetostrictive material, s_{kl} and s^μ_{kl} the total strain and magnetostriction strain tensors.

The integration of the piezomagnetic coefficients (4) between $\mathbf{S}^\mu$ and $\mathbf{S}$, leads to the magnetic behaviour law. This law can be written by introducing a coercive magnetic field $\mathbf{H}^c$ which describes the effect of the stress application:

$$\begin{aligned} h_i(\mathbf{B},\mathbf{S}) &= h^0_i(\mathbf{B},\mathbf{S}^\mu) - C^{ms}_{klnp}\frac{\partial s^\mu_{np}(\mathbf{B})}{\partial b_i}(s_{kl} - s^\mu_{kl}) \\ &= h^0_i(\mathbf{B},\mathbf{S}^\mu) - h^c_i(\mathbf{B},\mathbf{S}) \end{aligned} \qquad (7)$$

$h_i^c(\mathbf{B},\mathbf{S})$ is the magnetic field induced along i axis by stress at given magnetic flux density and $h_i^0(\mathbf{B},\mathbf{S}^\mu)$ is the magnetic field at free stress depending only of magnetic flux density.

B. Magnetostriction strain model

Magnetostriction strain is assumed isochoric and isotropic. In the reference frame of magnetic flux density $\left(b_\parallel, b_{\perp 1}, b_{\perp 2}\right)$, the magnetostriction strain tensor is diagonal and is expressed by a polynomial function of the magnetic induction (8):

$$s_\parallel^\mu(\mathbf{B}) = \sum_{n=0}^{N} \beta_n \;\|\mathbf{B}\|^{2(n+1)} \quad s_{\perp 1}^\mu = s_{\perp 2}^\mu = -\frac{s_\parallel^\mu(\mathbf{B})}{2} \tag{8}$$

where β_n are magnetostrictive coefficients identified from experimental magnetostriction curves[5] and $\|\mathbf{B}\|$ the norm of $\mathbf{B}$.To take into account the magnetic flux density distribution, it is necessary to consider the material frame. The magnetostriction strain tensor in this case is given by the following indicial form:

$$s_{kl}^\mu(\mathbf{B}) = \frac{1}{2}\sum_{n=0}^{N} \beta_n \;\|\mathbf{B}\|^{2n}\left(3b_k b_l - \delta_{kl}\,\|\mathbf{B}\|^2\right) \tag{9}$$

From the expression of the magnetostriction strain tensor, the coercive magnetic field (7) can be expressed as the product of an equivalent reluctivity tensor with the magnetic flux density:

$$h_i^c(\mathbf{B},\mathbf{S}) = \nu_{ij}^c(\mathbf{B},\mathbf{S})\; b_j \tag{10}$$

III. Finite element formulation

Considering the static case, the electro-magneto-mechanical problem is defined by a minimization of the functional energy $\mathcal{F}$ in terms of $\mathbf{E}$, $\mathbf{B}$ and $\mathbf{S}$:

$$\mathcal{F}(\mathbf{E},\mathbf{B},\mathbf{S}) = W(\mathbf{E},\mathbf{B},\mathbf{S}) - T \tag{11}$$

where $W(\mathbf{E},\mathbf{B},\mathbf{S})$ and T are respectively the electro-magneto-elastic energy and the work of electric, magnetic and mechanical sources. The minimization of (11) leads, after discretisation and assembling, to the algebraic equation system (12):

$$\begin{bmatrix} \mathbb{K}_{aa} & 0 & 0 \\ 0 & \mathbb{K}_{uu} & \mathbb{K}_{u\varphi} \\ 0 & \mathbb{K}_{\varphi u} & \mathbb{K}_{\varphi\varphi} \end{bmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{u} \\ \varphi \end{pmatrix} = \begin{pmatrix} \mathbf{J} + \mathbf{J}^c(\mathbf{B},\mathbf{S}) \\ \mathbf{F^T} + \mathbf{F}^{mf}(\mathbf{B}) + \mathbf{F}^\mu(\mathbf{B}) \\ \mathbf{Q} \end{pmatrix} \tag{12}$$

where $\mathbb{K}_{u\varphi} = -\mathbb{K}_{\varphi u}^t$ describes the electro-mechanical coupling, $\mathbf{J}^c(\mathbf{B},\mathbf{S})$ is an induced current density associated to the effect of stress on the magnetic behaviour, $\mathbf{F}^{mf}(\mathbf{B})$ and $\mathbf{F}^\mu(\mathbf{B})$ the equivalent nodal magnetic forces and magnetostriction forces respectively. $\mathbf{J}$ and $\mathbf{F^T}$ are the term due to external sources and $\mathbf{Q}$ the total electrical charge on the electrodes. Magnetic vector potential $\mathbf{a}$ is discretised with edge (3D) and nodal (2D) elements. The displacement $\mathbf{u}$ and the electric potential φ are discretised using nodal elements.

The numerical solution of this nonlinear problem is obtained by an iterative process, modified fixed point or Newton-Raphson procedure.

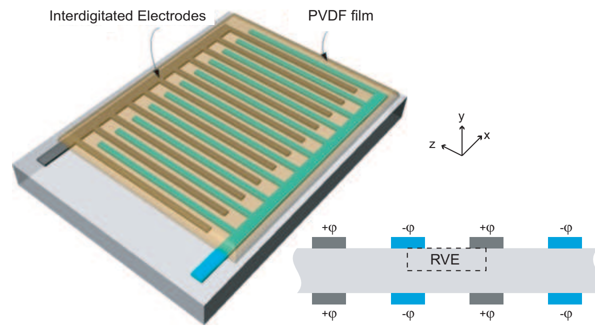

Fig. 1. The piezoelectric layer with interdigitated surface electrodes (RVE: Representative Volume Element)

IV. Results

The model is applied to a trilayer magnetoelectric composite, constituted by a piezolectric layer with interdigitated electrodes (Fig. 1) sandwiched between two magnetostrictive layers.

As shown in Fig.2, the electric field direction within the piezoelectric layer is non-uniform and will follow the field lines. Since the poling of the piezoelectric layer is performed using the interdigitated electrode patterns, the direction of poling will follow these field lines. Consequently, the piezoelectric material properties will continuously change with respect to the model axis. The numerical procedure involves, in a first step, an electrostatic computation to define the polarization orientation. According to this information the material properties of the piezoelectric layer are modified within each finite element.

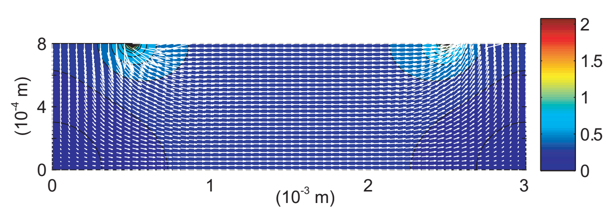

Fig. 2. Electric field (MV/m) between two electrodes fingers (RVE) in the xy plane

V. Conclusion

A magnetoelectric model, associated to a finite element formulation, has been presented. It is applied to specific structures for the design of magnetic sensors with interdigitated electrodes. In the full paper, the magnetoelectric effect model will be detailed as well as numerical results analysis.

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